TODCOR

Two-Dimensional Correlation

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The Cross-Correlation Function

Tonry & Davis 1979, *AJ*, **84,** 1511

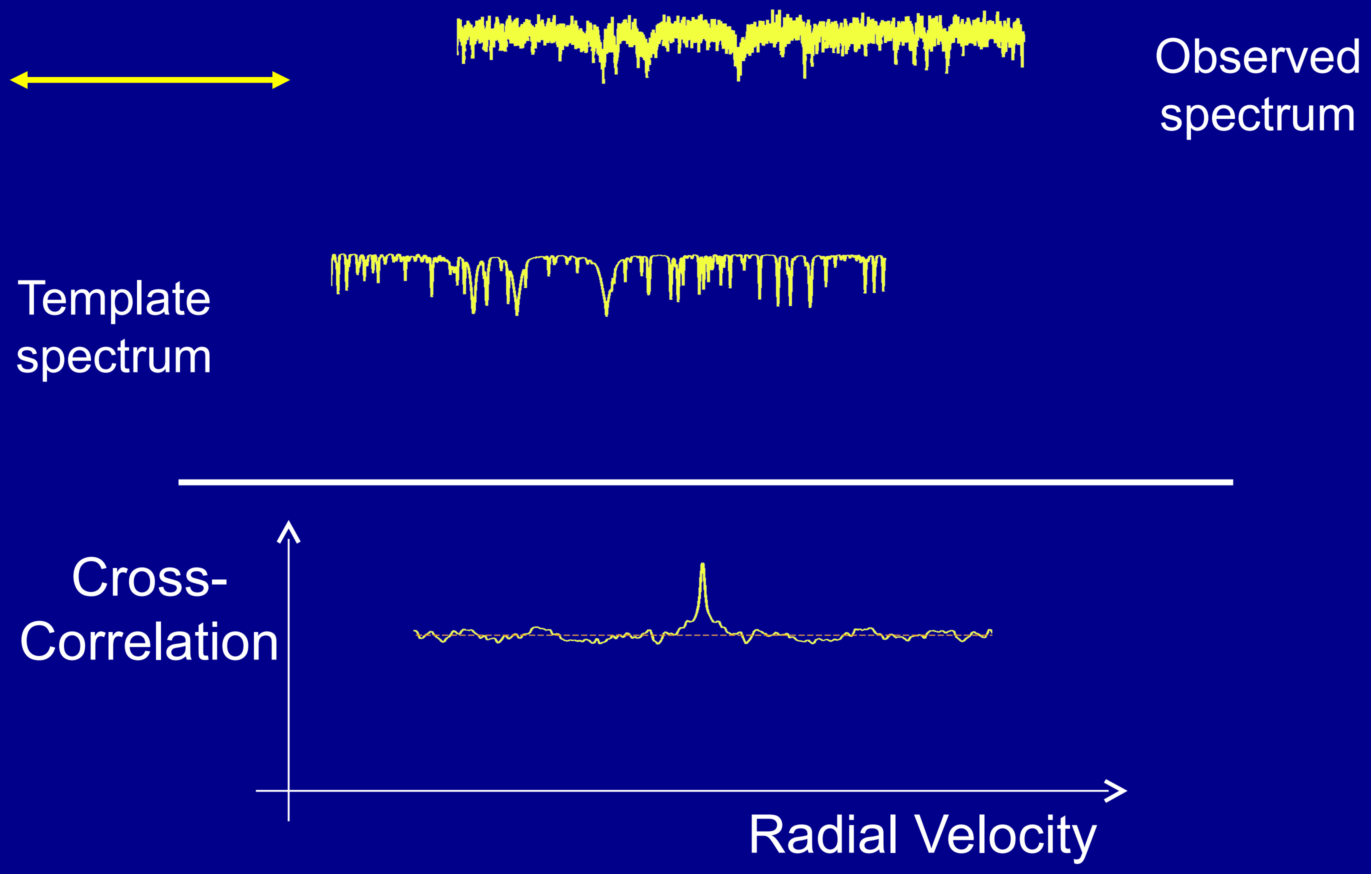
Wavelength-to-pixel	$n = A \ln \lambda + B$
Observed spectrum	$g(n)$
Template	$t(n)$

The Cross-Correlation Function

$$c(n) = \frac{1}{N\sigma_g\sigma_t}\sum_m g(m)\,t(m-n)$$

$$-1 \le c(n) \le 1$$

The Cross-Correlation Function



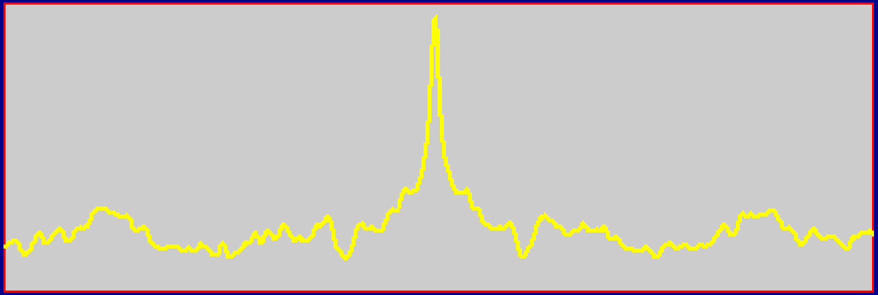
Observed spectrum

Template spectrum

Cross-Correlation

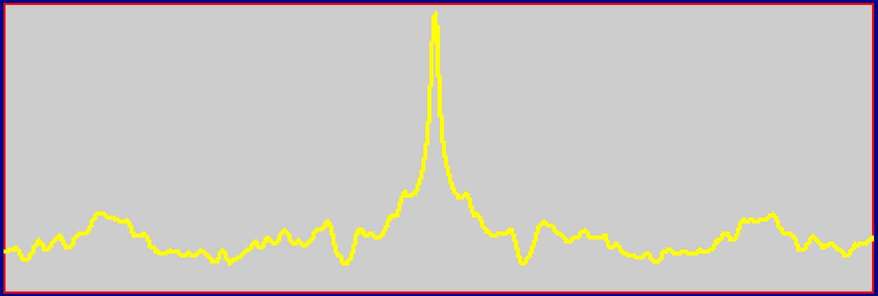
Radial Velocity

The Cross-Correlation Function



Single-line
Spectroscopic Binary

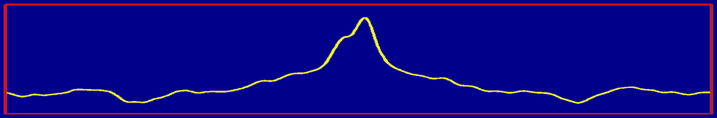
SB1



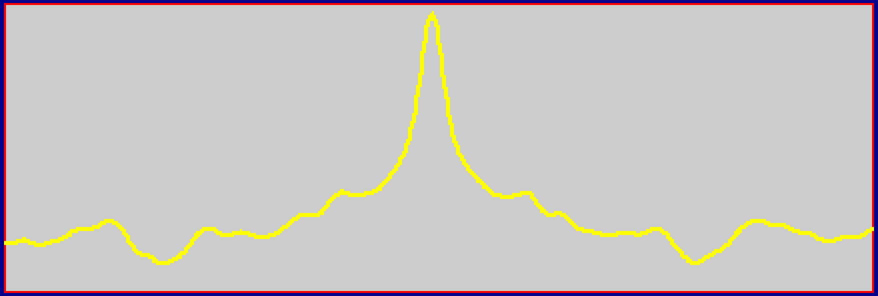
Double-line
Spectroscopic Binary

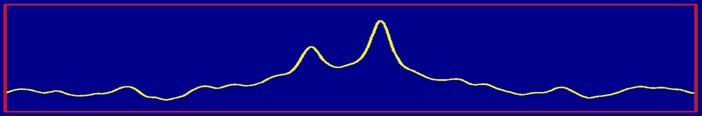
SB2

The Cross-Correlation Function



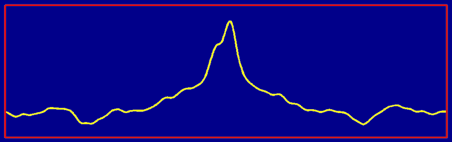
The two peaks
are blended



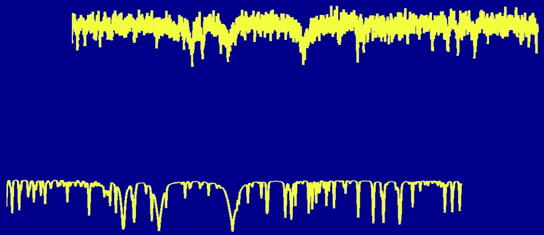


Two nicely
separated peaks

The Cross-Correlation Function



- Blended peaks
- Faint secondary spectrum
- Spectral mismatch between secondary and template



Two-Dimensinal CORrelation - Introduction

Zucker & Mazeh 1994, *ApJ*, **420**, 806

Observed spectrum	$g(n)$
Primary template	$t_1(n)$
Secondary template	$t_2(n)$
Relative weight	α

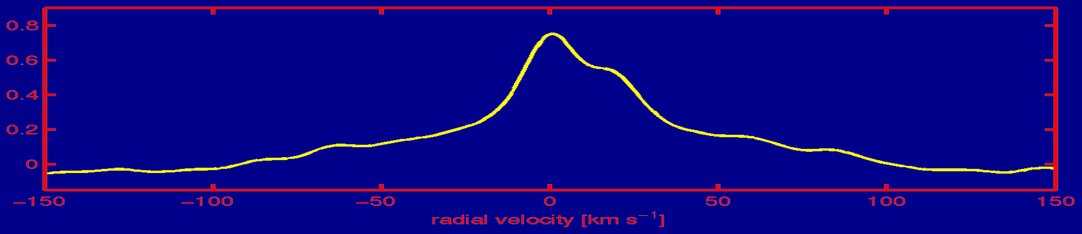
Two-Dimensinal CORrelation - Introduction

$$c(n) = \frac{1}{N\sigma_g \boxed{\sigma_t}} \sum_m g(m) \boxed{t(m-n)}$$

$$t(m-n) \longrightarrow t_1(m-n_1) + \alpha t_2(m-n_2)$$

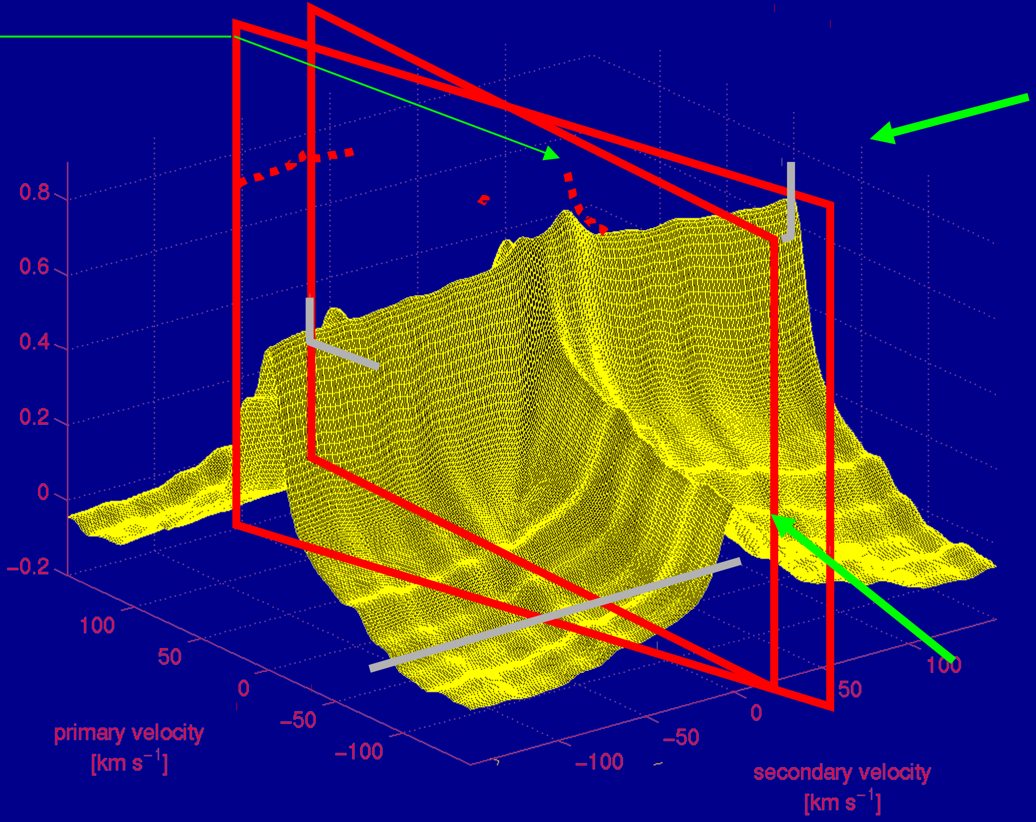
$$c(n_1, n_2; \alpha) = \frac{\boxed{c_1(n_1)} + \alpha \boxed{c_2(n_2)}}{\sqrt{1 + 2\alpha \boxed{c_{12}(n_2 - n_1)} + \alpha^2}}$$

Two-Dimensinal CORrelation - Introduction



Two-Dimensinal CORrelation - Introduction

Function of two variables



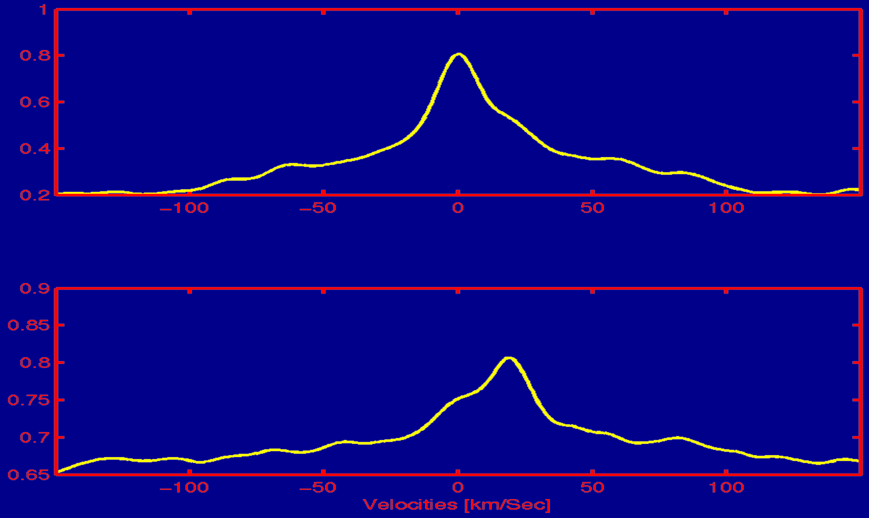
Estimated pair of velocities

Two main ridges

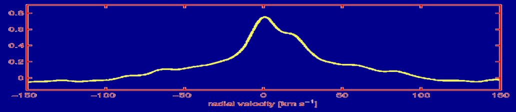
“Cuts” along the ridges (“freeze” one Velocity)

Two-Dimensinal CORrelation - Introduction

```
------------------- Results --------------------
    v1 =     0.0002 +-  0.1250
    v2 =   19.0979 +-  0.2578
    alpha =     0.5000 +-  0.0000
    Corr  =     0.8067
-----------------------------------------------------
```



Freeze the secondary velocity



Freeze the primary velocity

Estimate flux ratio

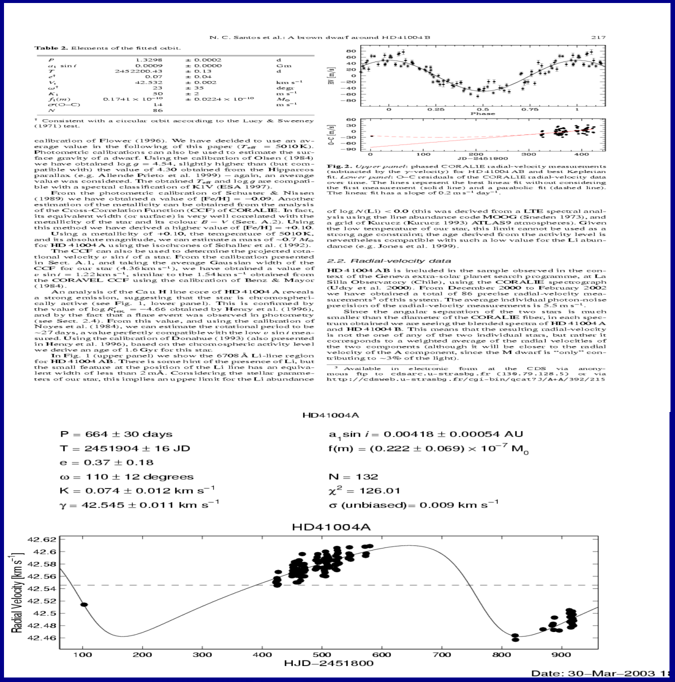
$$c(n_1,n_2;\alpha)=\frac{c_1(n_1)+\alpha c_2(n_2)}{\sqrt{1+2\alpha c_{12}(n_2-n_1)+\alpha^2}}$$

$$\hat{\alpha}(n_1,n_2)=\frac{c_1(n_1)c_{12}(n_2-n_1)-c_2(n_2)}{c_2(n_2)c_{12}(n_2-n_1)-c_1(n_1)}$$

$$R(n_1,n_2)=\sqrt{\frac{c_1^2(n_1)-2c_1(n_1)c_2(n_2)c_{12}(n_2-n_1)+c_2^2(n_2)}{1-c_{12}^2(n_2-n_1)}}$$

Multi-Order TODCOR

- In echelle spectra – the correlation from different orders has to be combined in some way (Zucker 2003, *MNRAS*, **342**, 1291)
- Different orders require different light ratios
- Introducing TODMOR
- Flagship case – HD41004



N. C. Santos et al.: A brown dwarf around HD 41004 B 217

Table 2. Elements of the fitted orbit.

P	1.3298	± 0.0002	d
$a_1 \sin i$	0.0009	± 0.0000	Gm
T	2452200.43	± 0.13	d
$e^{\dagger}$	0.07	± 0.04	
V_r	42.532	± 0.002	km s^{-1}
$\omega^{\dagger}$	23	± 35	degr
K_1	50	± 2	m s^{-1}
$f_1(m)$	0.1741×10^{-10}	$\pm 0.0224 \times 10^{-10}$	$M_{\odot}$
σ(O–C)	14		m s^{-1}
N	86		

† Consistent with a circular orbit according to the Lucy & Sweeney (1971) test.

calibration of Flower (1996). We have decided to use an average value in the following of this paper ($T_{\rm eff}$ = 5010 K). Photometric calibrations can also be used to estimate the surface gravity of a dwarf. Using the calibration of Olsen (1984) we have obtained $\log g = 4.54$, slightly higher than (but compatible with) the value of 4.30 obtained from the Hipparcos parallax (e.g. Allende Prieto et al. 1999) – again, an average value was considered. The obtained $T_{\rm eff}$ and $\log g$ are compatible with a spectral classification of K1V (ESA 1997).

From the photometric calibration of Schuster & Nissen (1989) we have obtained a value of [Fe/H] = −0.09. Another estimation of the metallicity can be obtained from the analysis of the Cross-Correlation Function (CCF) of CORALIE. In fact, its equivalent width (or surface) is very well correlated with the metallicity of the star and its colour $B-V$ (Sect. A.2). Using this method we have derived a higher value of [Fe/H] = +0.10.

Using a metallicity of +0.10, the temperature of 5010 K, and its absolute magnitude, we can estimate a mass of ~0.7 $M_{\odot}$ for HD 41004 A using the isochrones of Schaller et al. (1992).

The CCF can also be used to determine the projected rotational velocity $v \sin i$ of a star. From the calibration presented in Sect. A.1, and taking the average Gaussian width of the CCF for our star (4.36 km s^{-1}), we have obtained a value of $v \sin i$ = 1.22 km s^{-1}, similar to the 1.54 km s^{-1} obtained from the CORAVEL CCF using the calibration of Benz & Mayor (1984).

An analysis of the Ca II H line core of HD 41004 A reveals a strong emission, suggesting that the star is chromospherically active (see Fig. 1, lower panel). This is confirmed by the value of $\log R'_{\rm HK} = -4.66$ obtained by Henry et al. (1996), and by the fact that a flare event was observed in photometry (see Sect. 2.4). From this value, and using the calibration of Noyes et al. (1984), we can estimate the rotational period to be ~27 days, a value perfectly compatible with the low $v \sin i$ measured. Using the calibration of Donahue (1993) (also presented in Henry et al. 1996), based on the chromospheric activity level we derive an age of 1.6 Gyr for this star.

In Fig. 1 (upper panel) we show the 6708 Å Li-line region for HD 41004 AB. There is some hint of the presence of Li, but the small feature at the position of the Li line has an equivalent width of less than 2 mÅ. Considering the stellar parameters of our star, this implies an upper limit for the Li abundance of $\log N(\rm Li) < 0.0$ (this was derived from a LTE spectral analysis using the line abundance code MOOG (Sneden 1973), and a grid of Kurucz (Kurucz 1993) ATLAS9 atmospheres). Given the low temperature of our star, this limit cannot be used as a strong age constraint; the age derived from the activity level is nevertheless compatible with such a low value for the Li abundance (e.g. Jones et al. 1999).

Fig. 2. *Upper panel*: phased CORALIE radial-velocity measurements (subtracted by the γ-velocity) for HD 41004 AB and best Keplerian fit. *Lower panel*: O–C residuals of the CORALIE radial-velocity data over time. The lines represent the best linear fit without considering the first measurement (solid line) and a parabolic fit (dashed line). The linear fit has a slope of 0.2 m s^{-1} day^{-1}.

2.2. Radial-velocity data

HD 41004 AB is included in the sample observed in the context of the Geneva extra-solar planet search programme, at La Silla Observatory (Chile), using the CORALIE spectrograph (Udry et al. 2000). From December 2000 to February 2002 we have obtained a total of 86 precise radial-velocity measurements[3] of this system. The average individual photon-noise precision of the radial-velocity measurements is 5.5 m s^{-1}.

Since the angular separation of the two stars is much smaller than the diameter of the CORALIE fiber, in each spectrum obtained we are seeing the blended spectra of HD 41004 A and HD 41004 B. This means that the resulting radial-velocity is not the one of any of the two individual stars, but rather it corresponds to a weighted average of the radial velocities of the two components (although it will be closer to the radial velocity of the A component, since the M dwarf is "only" contributing to ~3% of the light).

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[3] Available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/392/215

HD41004A

P = 664 ± 30 days
T = 2451904 ± 16 JD
e = 0.37 ± 0.18
ω = 110 ± 12 degrees
K = 0.074 ± 0.012 km s^{-1}
γ = 42.545 ± 0.011 km s^{-1}

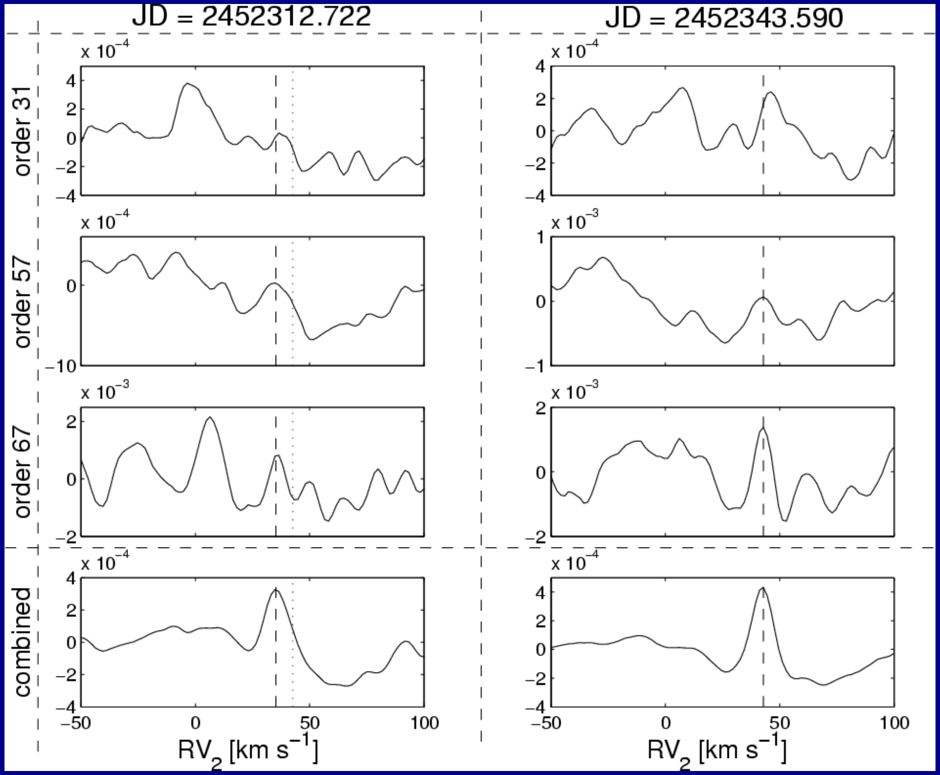
$a_1 \sin i$ = 0.00418 ± 0.00054 AU
f(m) = (0.222 ± 0.069) × 10^{-7} M$_0$

N = 132
χ^2 = 126.01
σ (unbiased)= 0.009 km s^{-1}

HD41004A

Date: 30–Mar–2003 1

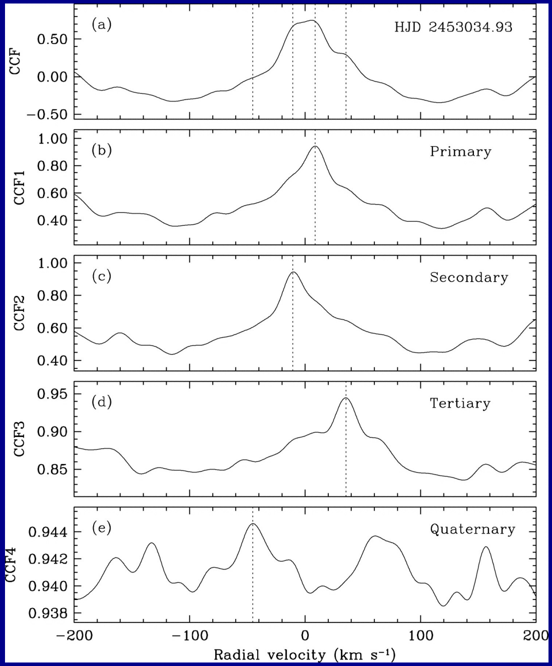
Multi-Order TODCOR

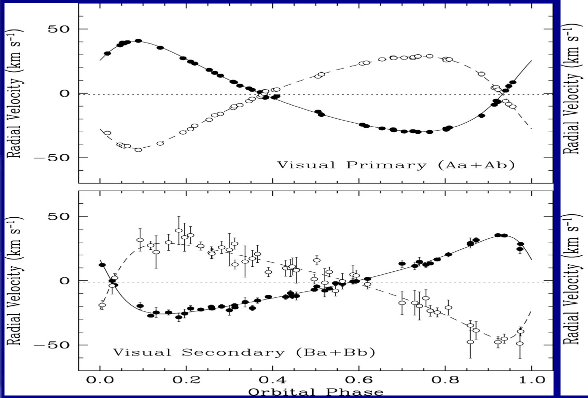


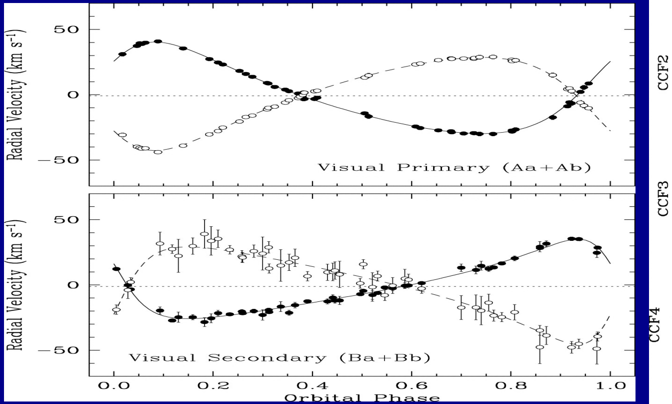
Zucker et al. 2003, *A&A*, **392**, 215

Generalize TODCOR to triples (and quadruples)

- TRICOR – 3D correlation
  (Zucker, Torres & Mazeh 1995, *ApJ*, **452**, 863)
- 'QUADCOR' - HD110555
  (Torres, Latham & Stefanik 2007, *ApJ*, **662**, 602)

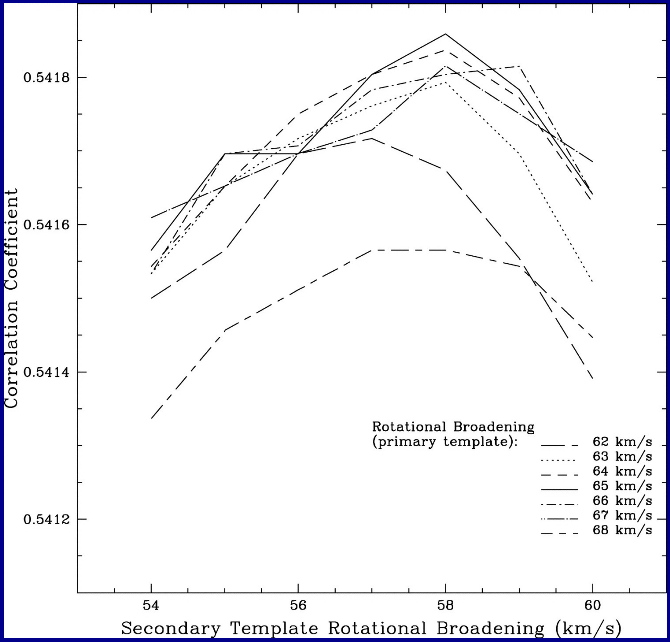




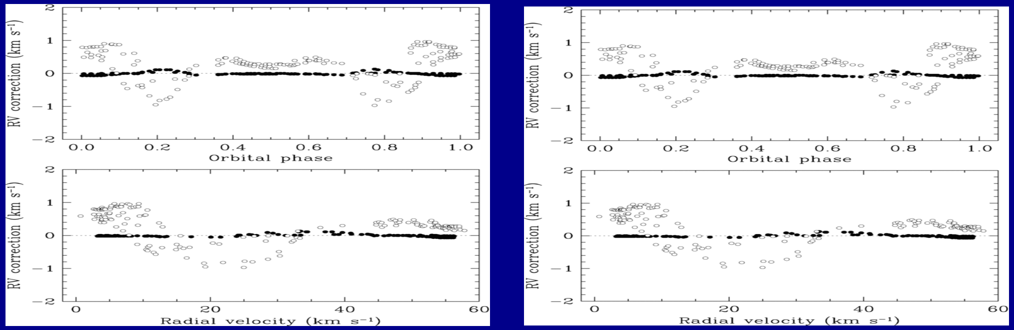


Common Practice

- Enumerate over a grid of templates
- Templates are either synthetic or high S/N spectra
- Optimize over the grid to find the highest correlation.
- Residual systematics can be accounted for by simulations



López-Morales & Ribas 2005, *ApJ*, **631**, 1120 (GU Boo)

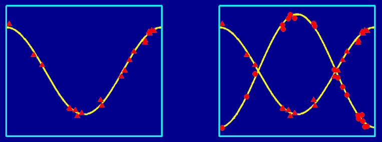


Torres, Claret & Young 2009, *ApJ*, **700**, 1349 (Capella)

TODCOR vs. Spectral disentangling

TODCOR	SD
Estimates 2-3 variables	Estimates $O(103)$-$O(102)$
Analyzes individual exposures	Needs good coverage of orbit
Outputs only RV	Outputs spectra and RV
Low S/N	High S/N
Good known spectra	No need to know spectra

Application: study mass-ratio distribution

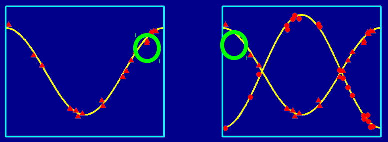


SB1

$$f(M_2) = M_1 \frac{(q \sin i)^3}{(1+q)^2}$$

$q_{\min}$ (assuming $\sin i = 1$)

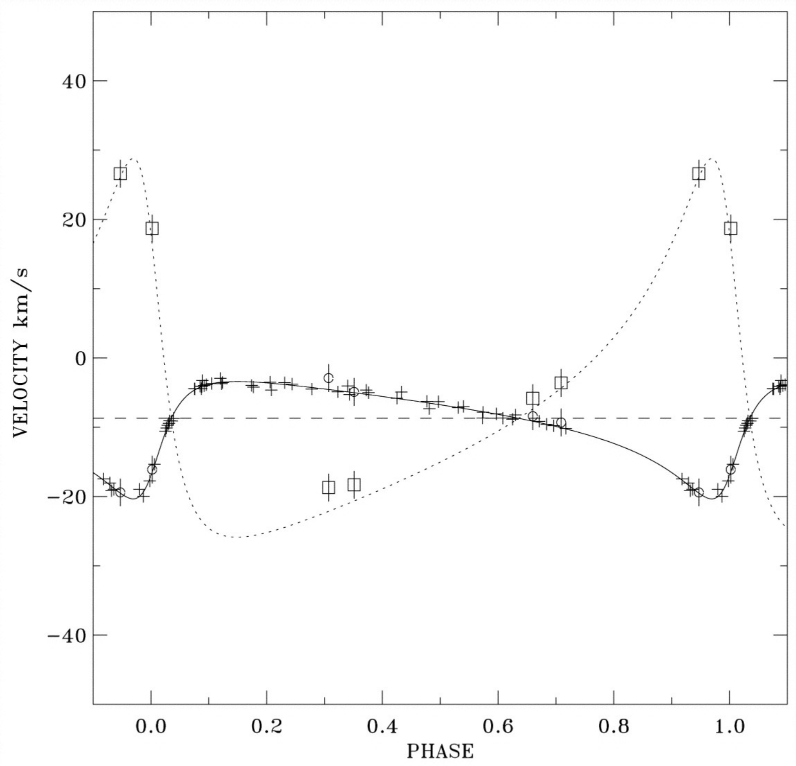
$$q = \frac{K_1}{K_2}$$



SB2

Application: study mass-ratio distribution

Haro 1-14c



Simon & Prato 2004, *ApJ*, **613**, L69

Applications for extrasolar planets

- Remove contamination from sky/moon
- Test for stellar nature of candidate planet

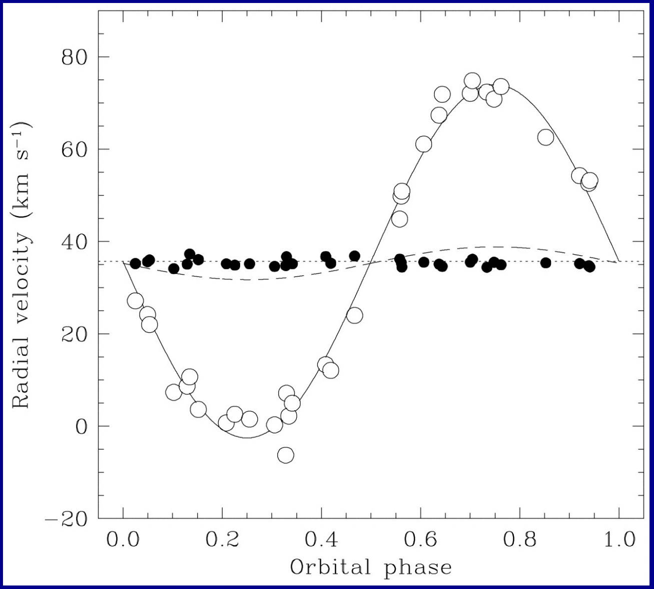
Konacki et al. 2003, *Nature*, **421**, 507 (OGLE-TR-56)

- Test blend scenarios for transits

Torres et al. 2004, *ApJ*, **614**, 979

- Search for planets around SB2s

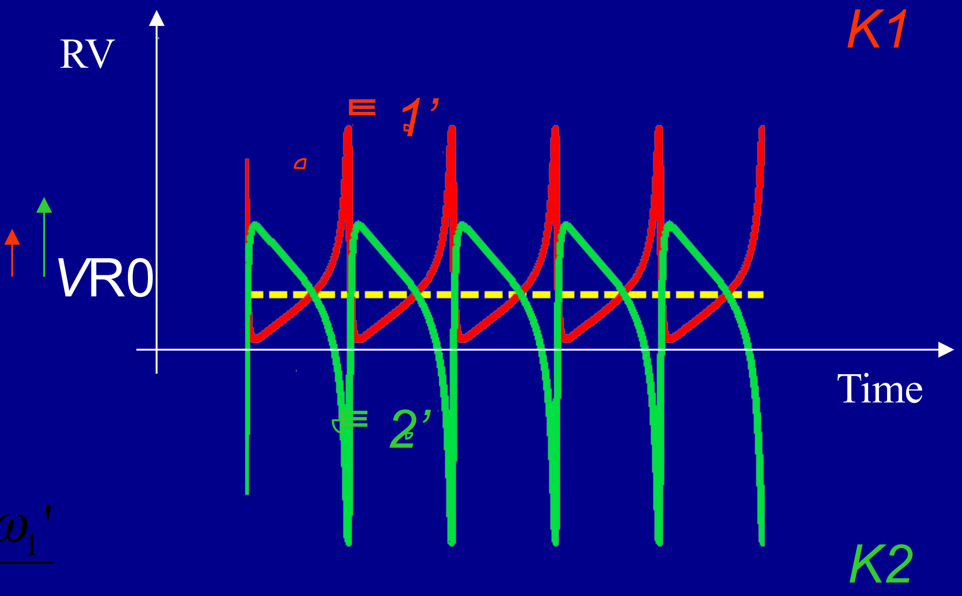
Konacki et al. 2010, *ApJ*, **719**, 1293



Mandushev et al. 2005,
ApJ, **621**, 1061
(GSC 01944-02289)

Possible future application: relativistic effects

The apparent difference between ≡ `1 and ≡ `2 is a relativistic effect



$$\sin^2 i = \frac{3e}{\omega_2'-\omega_1'}\frac{K_2'\sin\omega_2'+K_1'\sin\omega_1'}{c}$$

Zucker & Alexander 2007, *ApJ*, **654**, L83

Konacki et al. 2010, *ApJ*, **719**, 1293 (12 Boo)

A full direct measurement of the stellar masses