

# SCD Calibration

Steven Tomczyk, Alfred deWijn  
and the SCD Team

# Objective

Polarimetric calibration of the SCD Instrument

This formulation is based on:

seminal paper by del Toro Iniesta and Collados (dTI&C),  
“Optimum modulation and demodulation matrices for  
solar polarimetry”, App. Opt., 39, 1637, 2000

and presentation by deWijn on “The Concept of Optimal  
Calibration”

The notation of dTI&C will be used here. The matrix  
dimensions are row x column

# Theory

- The measurement process is defined by  $I_{\text{meas}} = O I_{\text{in}}$
- $I_{\text{in}}$  is the Stokes vector (I,Q,U,V) input to the polarimeter (4 vector)
- $O$  is the modulation matrix ( $n \times 4$  matrix)
- $n$  is the number of modulation states
- $I_{\text{meas}}$  is the vector of measured intensities ( $n$  vector)
- For SCD,  $n = 4$

# Theory

- The goal is to obtain  $I_{in}$  from  $I_{meas}$
- This process is called demodulation
- We can define  $I_{in} = D I_{meas}$
- Where  $D$  is the demodulation matrix ( $4 \times n$ )
- dTI&C find that it is not feasible in general to obtain  $D$  by taking the inverse of  $O$

# Theory

- Instead dTI&C compute  $D$  from  $O$  by first computing the matrix  $A = O^T O$  (4 x 4 matrix)
- Then  $D = A^{-1} O^T = (O^T O)^{-1} O^T$
- The matrix  $A$  also contains valuable information about the efficiency of the polarimeter which relates to the noise on the demodulated Stokes vector (see dTI&C)
- The efficiencies are computed from the diagonal elements of  $A$  by  $e_i = 1/(nA^{-1}_{i,i})$

# Calibration

- Computation of  $D$  requires knowledge of  $O$  which is obtained by the calibration process
- For SCD calibration, we will assume that we know the properties (Mueller matrix vs.  $\lambda$ ) of the calibration optics
- The  $\lambda$  dependence is important since the SCD was designed to have high polarimetric efficiency over the entire wavelength range, but  $D$  varies greatly with  $\lambda$  (see Tomczyk et al, JOSA, 2010)
- We will also (initially) assume that the light entering the polarimeter is unpolarized with Stokes vector  $[1,0,0,0]$

# Calibration

- Following deWijn we can draw an analogy to the dTI&C procedure and define a calibration matrix
- $C = [I_{in,1} \ I_{in,2} \ \dots \ I_{in,m}]$ , (4 x m matrix) where m is the number of calibration states and  $I_{in,j}$  is the Stokes vector produced by calibration state j (for SCD, m = 6)
- $I_{in,j} = MM_j I_C$ , where  $I_C$  is the Stokes vector of light entering the calibration optics and  $MM_j$  is the Mueller matrix of calibration state j (we are assuming  $I_C = [1,0,0,0]$  initially)
- The m sets of measured intensities for the calibration are then  $I_{meas,j} = O I_{in,j}$  or  $I_{meas \ n,m} = O C$

# Calibration

- where  $I_{\text{meas } n,m}$  is a  $n$  row by  $m$  column matrix containing the  $n$  intensities for the  $m$  calibration states
- deWijn defines a matrix  $E$  ( $m \times 4$ ) such that
$$O = I_{\text{meas } n,m} E$$
- then  $O = O C E$  and  $C E = 1$ .
- $E$  is the nominal inverse of  $C$  computed by
$$E = C^T (C C^T)^{-1}$$
(the diagonal terms of  $C C^T$  provide calibration efficiencies)
- and  $O$  can be computed by  $O = I_{\text{meas } n,m} E$

# Calibration Procedure

1. create  $C = [I_{in,1} \ I_{in,2} \ \dots \ I_{in,m}]$  matrix ( $4 \times m$ ) from the calibration optics Mueller matrices using  $I_{in,j} = MM_j I_C$ , assuming  $I_C = [1, 0, 0, 0]$
2. compute the  $E$  matrix ( $m \times 4$ ) from the  $C$  matrix using  $E = C^T (C C^T)^{-1}$
3. create the  $I_{meas\ n,m}$  matrix ( $n \times m$ ) from the calibration observations comprised of the  $n$  intensities for the  $m$  calibration states (after bias and flat correction)
4. compute the  $O$  matrix ( $n \times 4$ ) from  $O = I_{meas\ n,m} E$
5. compute  $A = O^T O$ , then  $D = A^{-1} O^T$

# Consistency Check

- It is possible to check the consistency of the calibrations using the additional clear observed as part of the calibration scheme
- Apply the D matrix to the clear observation
- It should yield a Stokes vector very close to  $[1,0,0,0]$
- If it does not, substitute the clear Stokes vector for  $I_c$  , go back to step 1 and recompute D
- Iterate if needed

# Apply

- Now that  $D$  has been computed, it can be applied to the measurements
- Bias subtract and flat field divide the intensity vector  $I_{\text{meas}}$  at each pixel
- Apply the  $D$  matrix to obtain the Stokes vector
- Note that  $D$  may vary across the image and may need to be computed at various spatial points and interpolated
- $D$  will need to be computed for each wavelength region but should not vary appreciably across a spectral line

# Next Steps

- I recommend that 1 main program and 5 subroutines be written to perform SCD calibration
- The 5 subroutines will perform the 5 corresponding tasks on the Calibration Procedure slide
- Steve will write:
  - SCD\_Create\_Cmatrix
  - SCD\_Compute\_Ematrix
  - SCD\_Compute\_Omatrix
  - SCD\_Compute\_Dmatrix
  - That correspond to steps 1, 2, 4 and 5 respectively
- AISAS will write SCD\_Get\_Cal\_Data that will perform step 3

# Next Steps

- AISAS will write the SCD\_Calibration main program that will call all of these routines
- My goal is to have a draft of the 4 routines by the end of next week
- All of this is up for discussion
- Comments and feedback are very welcome
- It is strongly recommended that all involved become very familiar with the dTI&C paper