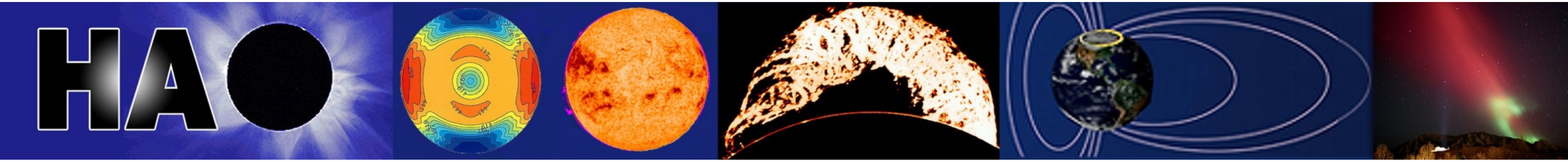


# The Concept of Optimal Calibration

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# Optimal Modulation

- 2000ApOpt..39.1637D
- Introduces the concept of the efficiency of a modulation scheme.
- Led to the development of the "polychromatic modulator" that is efficient at all wavelengths but (possibly highly) chromatic. DKIST uses these for modulation.

# Optimal Calibration

- Can we define a “calibration efficiency” the same way we defined the “modulation efficiency”?
- There is some hope that we can – calibration optics are kind of like inverted modulation optics.

# Optimal Calibration

- Short answer: yes.
- Good news: completely analogous to modulation efficiency!
- Those with jetlag may catch up on some sleep now.

# Math: Definitions

- $\mathbf{O}$ is the $n \times 4$ modulation matrix (the first rows of the polarimeter Mueller matrix for each of the $n$ states)
- Then the optimal demodulation matrix is $\mathbf{D} = (\mathbf{O}^{\mathrm{T}}\,\mathbf{O})^{-1}\,\mathbf{O}^{\mathrm{T}}$
- Efficiencies are $\epsilon_i^2 = 1/(n\,(\mathbf{O}^{\mathrm{T}}\,\mathbf{O})^{-1}{}_{ii})$.
- Errors in the Stokes vector scale with $1/\epsilon_{\mathrm{i}}$.

# Math: Calibration Matrix

- Introduce the $4 \times m$ calibration matrix: $\mathbf{C} = (\mathbf{I}_{\mathrm{in},1}\ \mathbf{I}_{\mathrm{in},2}\ \ldots\ \mathbf{I}_{\mathrm{in},m})$
- $\mathbf{I}_{\mathrm{meas}} = \mathbf{O}\ \mathbf{C}$
- Now assume we have an $\mathbf{E}$ such that $\mathbf{O} = \mathbf{I}_{\mathrm{meas}}\ \mathbf{E} = \mathbf{O}\ \mathbf{C}\ \mathbf{E}$
- $\mathbf{E}$ exists, $\mathbf{C}\ \mathbf{E} = \mathbb{1}$, and $\mathbf{E} = \mathbf{C}^{\mathrm{T}}\ (\mathbf{C}\ \mathbf{C}^{\mathrm{T}})^{-1}$.

# Math: Error Propagation

- Assume every measurement in $\mathbf{I}_{\mathrm{meas}}$ has an error $\sigma$. (This isn't true.)
- $\sigma_j^2 = \sigma^2 \sum_k E_{kj}^2$
- Analogously to modulation efficiency, we can now define calibration efficiency as: $\eta_j = (m \sum_k E_{kj}^2)^{-1/2}$
- And we see also that $\underline{\sigma}_j^2 = \sigma^2 / \eta_j^2$

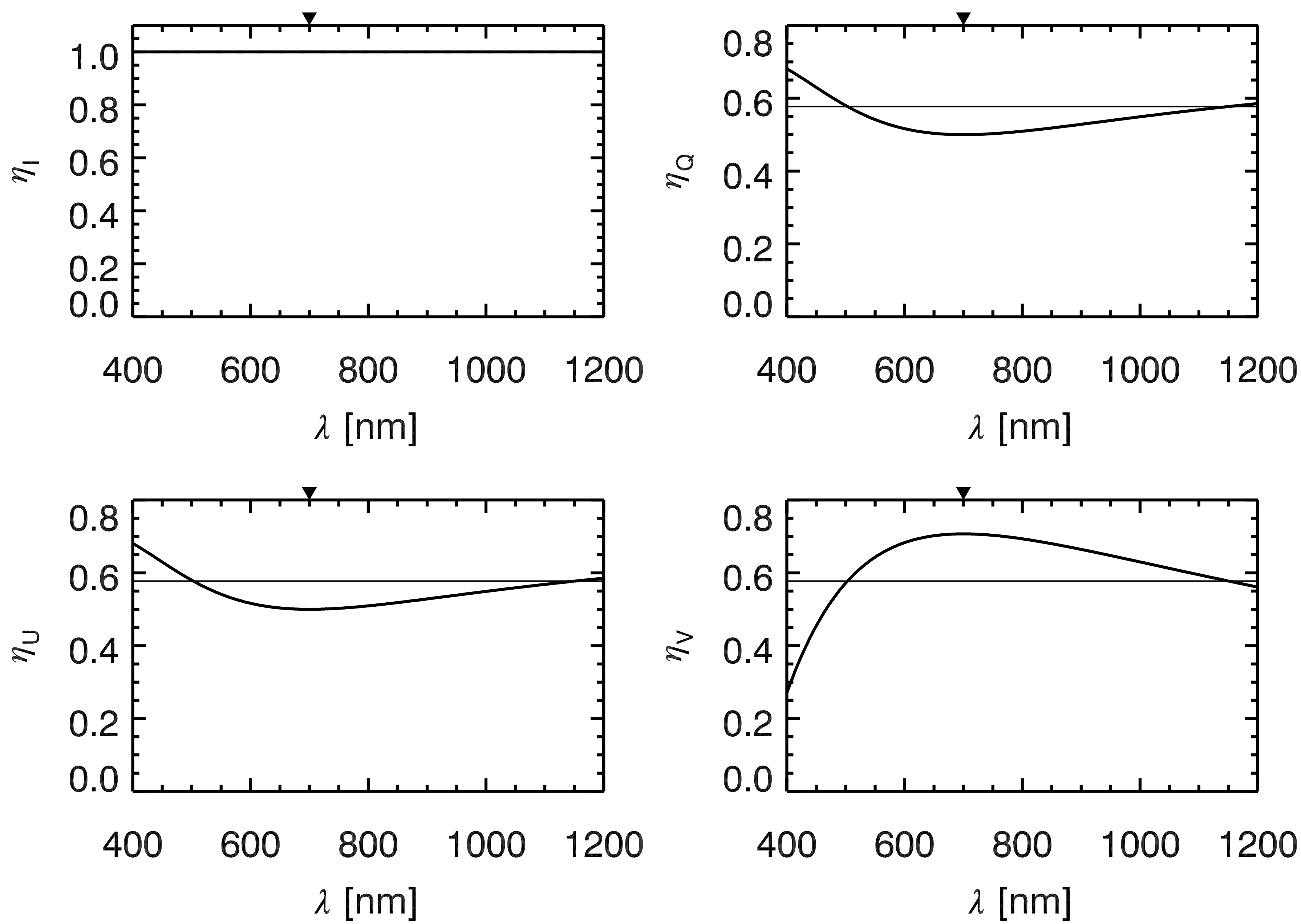
# Optimal Calibration

- Notice that this is fully analogous to modulation efficiency. Hence we can follow JCdTI & MC.
- Define $\mathbf{B} = (\mathbf{C}\,\mathbf{C}^{\mathrm{T}})$.
- $\eta_i^2 = 1/(m\, B_{ii})$
- A more diagonal $\mathbf{B}$ corresponds to a more optimal calibration scheme.

# Practical Example

- Calibration package consisting of a perfect polarizer and a perfect ¼-wave retarder at 700 nm.
- Input Stokes vector is unpolarized.
- The retarder fast axis is rotated to 0, 45, 90, and 135 degrees for each of the polarizer positions of 0, 45, 90, and 135 degrees.

# Practical Example



# Questions and Ideas

- The errors on the components of $\mathbf{I}_{\mathrm{meas}}$ are not all the same. What is the consequence?
- We usually don't know what $\mathbf{C}$ is. What is the consequence?
- Can the $\mathbf{E}$ matrix formulation help in the reduction of calibration data for DKIST?
- Can we deterministically find $\mathbf{O}$?